

Spin-Seebeck effect in a strongly interacting Fermi gas

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We study the spin-Seebeck effect in a strongly interacting, two-component Fermi gas and propose an experiment to measure this effect by relatively displacing spin up and spin down atomic clouds in a trap using spin-dependent temperature gradients. We compute the spin-Seebeck coefficient and related spin-heat transport coefficients as functions of temperature and interaction strength. We find that when the inter-spin scattering length becomes larger than the Fermi wavelength, the spin-Seebeck coefficient changes sign as a function of temperature, and hence so does the direction of the spin-separation. We compute this zero-crossing temperature as a function of interaction strength and in particular in the unitary limit for the inter-spin scattering.

Introduction.— Spin caloritronics, the study of coupled spin and heat transport, is a rapidly developing subfield of spintronics [1]. In particular, the spin-dependent generalization of the Seebeck effect, called the spin-Seebeck effect, has been intensively studied in the solid-state environment [2]. Recently, there has been broad interest in exploring spintronic phenomena in cold atomic systems [3–5]. Spin transport in a strongly interacting, two component Fermi gas was investigated experimentally in Ref. [6]. It is the purpose of this Letter to study the associated heat transport, i.e., thermo-spin effects, in a similar setting.

In the ordinary Seebeck effect in metals, an electrochemical potential gradient is generated by applying a temperature gradient. Similarly, for a gas with two spin states, the spin-Seebeck coefficient S_s determines the spin chemical potential μ_s generated by a spin temperature gradient ∇T_s through the relation $\nabla \mu_s = S_s \nabla T_s$, where $\mu_s \equiv \mu_+ - \mu_-$ and $T_s \equiv T_+ - T_-$, μ_σ and T_σ being the spin-dependent chemical potential and temperatures of the spin σ atoms, respectively, and we label spin components by $+$ and $-$. To measure the spin-Seebeck coefficient, we propose relatively displacing the center of mass of spin up and spin down atom clouds in a harmonic trap by applying a spin-dependent temperature gradient, for example by selectively heating one spin component with a laser, as illustrated in Fig. 1. The locations x_\pm of the center of mass of the spin up and down atoms are shifted to the minimum of $\mu_\pm + V$, where V trapping potential, resulting in a spin separation $x_s = x_+ - x_- = S_s \nabla T_s / m \omega^2$, where m is the mass of the atoms, ω is the trap frequency in the direction of the temperature gradients. For an order of magnitude estimate, we take $S_s \simeq .01 k_B$, as verified below. For $\nabla T_s = 10^{-5}$ K/cm [7], $\omega = 2\pi \times 1.46$, we find $x_s \simeq 1$ mm, which is well within experimental resolution.

We have computed the spin-Seebeck coefficient for a two-component Fermi gas, plotted in Fig. 2, as a function of temperature and for several values of the interaction strength $k_F a$, where k_F is the Fermi wave vector and a is the inter-spin scattering length. As seen from the figure, for weak interactions ($k_F a < 1$), S_s is small and negative, while for strong interactions ($k_F a \geq 1$), S_s is

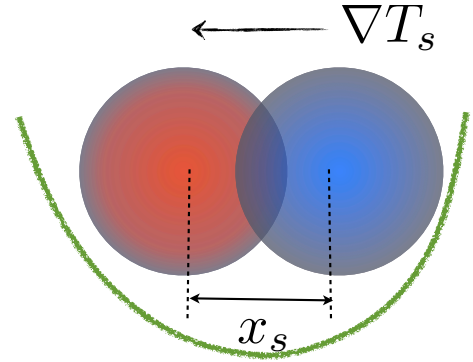


FIG. 1: Spin up and spin down atomic clouds are spatially relatively displaced in the presence of a spin temperature gradient. The distance between the center of mass of the different spin components, denoted by x_s , is proportional to the spin-Seebeck coefficient.

larger and its sign changes as a function of temperature. In terms of the experiment mentioned above, this means that the spin displacement changes direction as a function of temperature, which is an interesting qualitative effect. We also plot the zeros of S_s as a function of $k_F a$ in Fig. 3d. The temperature of the zero-crossing reaches a universal value T_0 , in the unitary limit for the inter-spin scattering length $k_F a \rightarrow \infty$, We find $T_0 \simeq .378 T_F$ in our calculation, where T_F is the Fermi temperature.

The thermodynamic reciprocal of the spin-Seebeck effect is the spin-Peltier effect, in which a spin-dependent heat current proportional to the spin-Seebeck coefficient is induced by a spin current. This effect will heat up spin-up and spin-down components differently and provides another way to measure S_s . Furthermore, as discussed in Ref. [5], the spin-Seebeck effect contributes to the total dissipation so that S_s can also be measured through the heating.

We note that the spin-Seebeck coefficient was calculated for a weakly interacting Bose gas in Ref. [5], but the Bose gas is unstable towards the formation of molecules for large scattering lengths, which makes the strongly interacting regime more difficult to realize experimentally.

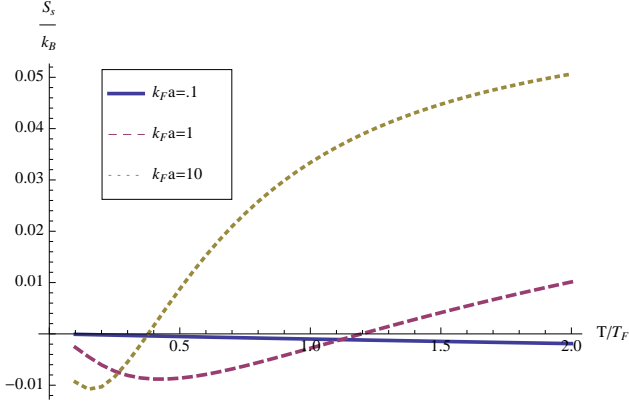


FIG. 2: The Seebeck coefficient plotted as a function of the reduced temperature T/T_F , where T_F is the Fermi temperature, for values of $k_F a$ representing weak ($k_F a = .1$) and strong ($k_F a = 1$) coupling, and approaching the unitary limit ($k_F a = 10$).

Phenomenology.— We are specifically interested in phenomena due to *spindrag*, the transfer of momentum between different spins due to inter-spin scattering, which allows one to generate currents in one spin species by applying forces on the other, and we define a set of spin-heat transport coefficients which captures these effects as follows. We consider a Fermi gas with two different spin states selected from a larger half-integer spin multiplet, which we will call “spin up” (+) and “spin down” (−), for equal spin up/down densities $n_+ = n_- \equiv n$, i.e., in the absence of spin polarization, and apply equal and opposite forces and temperature gradients for the two spin species, i.e., $\mathbf{F}_+ = -\mathbf{F}_-$ and $\nabla T_+ = -\nabla T_-$. In linear response, the ensuing spin current and spin heat current defined by $\mathbf{j}_s = \mathbf{j}_+ - \mathbf{j}_-$, and $\mathbf{q}_s = \mathbf{q}_+ - \mathbf{q}_-$, respectively, are given by

$$\begin{pmatrix} \mathbf{j}_s \\ \mathbf{q}_s \end{pmatrix} = \sigma_s \begin{pmatrix} 1 & S_s \\ T S_s & \kappa_s (1 + Z_s T) \end{pmatrix} \begin{pmatrix} \mathbf{F}_s \\ -\nabla T_s \end{pmatrix}, \quad (1)$$

where $\mathbf{F}_s \equiv \mathbf{F}_+ - \mathbf{F}_-$ is the spin force, $T_s = T_+ - T_-$ is the spin temperature, T is the equilibrium temperature, σ_s is the spin conductivity, κ_s is the spin heat conductivity (at zero spin current), $Z_s T = \sigma_s S_s^2 T / \kappa_s$, and Onsager reciprocity is explicitly included in the matrix above. We note that \mathbf{F}_s is the thermodynamic force which includes forces coming from pressure gradients, i.e., $\mathbf{F}_s = \mathbf{f}_s^{\text{ext}} - \nabla p_s / n$, where $\mathbf{f}_s^{\text{ext}}$ is the external spin force, and $p_s = p_+ - p_-$ is the difference in pressures of the spin up and down atoms. These coefficients, computed with the Boltzmann equation described below, are plotted in Fig. 3 as functions of T/T_F for several values of $k_F a$.

As is well known, the spin conductivity σ_s rapidly increases at low temperatures due to Pauli blocking. Our result for σ_s , plotted in Fig. 3a includes corrections due to spin-heat coupling, but they are negligibly small, so

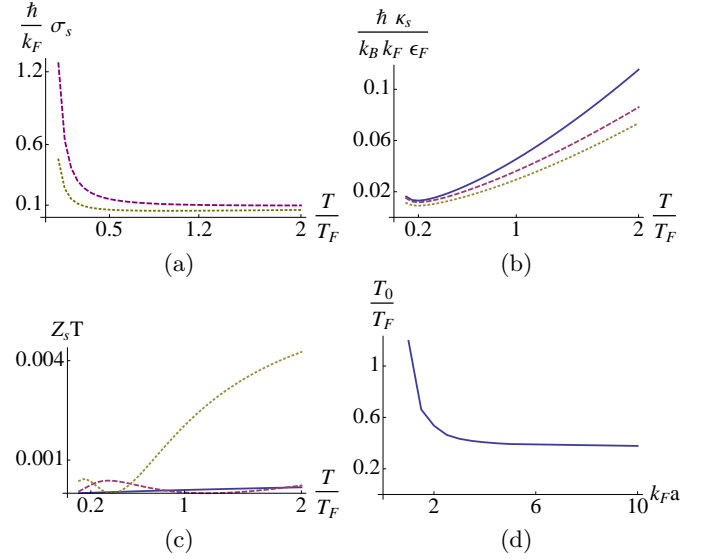


FIG. 3: (a) A plot of the spin conductivity normalized as $\hbar \Lambda \sigma_s$, (b) the spin heat conductivity normalized as $\hbar \Lambda \kappa_s / k_B \epsilon_F$, and (c) the figure of merit $Z_s T$, for $k_F a = .1$ (thick blue), $k_F a = 1$ (dashed purple), $k_F a = 10$ (dotted yellow). (d) A plot of the temperatures T_0 relative to T_F where $S_s = 0$ as a function of $k_F a$.

that one can safely take $\sigma_s = n \tau_s / m$ with τ_s the spindrag relaxation time [9] measured in Ref. [6] and calculated in Ref. [8]. The downturn of S_s at low temperatures is a quantum mechanical effect that also occurs for bosons, where in contrast to fermions, the spin conductivity decreases sharply at low temperatures due to bosonic enhancement of scattering [10]. The spin heat conductivity κ_s is plotted in Fig. 3b, where it is seen to increase with increasing T . The dimensionless figure of merit $Z_s T$ (Fig. 3c) determines the thermodynamic efficiency of engines based on thermo-spin effects [11].

A spin-dependent temperature gradient can only be established when intra-spin scattering is much greater than inter-spin scattering. For fermions, the first nonvanishing intra-spin scattering amplitude is p -wave. To make this large, one can tune the p -wave scattering length by a Feshbach resonance [12]. Taking the unitary limit for intra-spin scattering, the intra and inter-spin differential cross section are given by

$$\begin{aligned} \frac{d\sigma_{++}}{d\Omega} &= \frac{d\sigma_{--}}{d\Omega} = \frac{9(\hat{\mathbf{p}}_r \cdot \hat{\mathbf{p}}'_r)^2}{(p_r/\hbar)^2}, \\ \frac{d\sigma_{+-}}{d\Omega} &= \frac{a^2}{1 + (p_r a/\hbar)^2}, \end{aligned} \quad (2)$$

where $p_r = |\mathbf{p}_r|$ is the relative momentum of incoming particles with momenta \mathbf{p}_1 and \mathbf{p}_2 , defined by $\mathbf{p}_r = (\mathbf{p}_1 - \mathbf{p}_2)/2$, the hat superscripts denotes unit vectors, and a is the inter-spin s -wave scattering length.

Calculation of transport coefficients.— Next, we present the computation of S_s using the Boltzmann equation.

We parametrize the non-equilibrium, steady-state distribution by

$$n_{\mathbf{p}\sigma}(\mathbf{r}) = f_{\mathbf{p}\sigma}(\mathbf{r}) - \partial_\epsilon f_{\mathbf{p}}^0 \phi_{\mathbf{p}\sigma}(\mathbf{r}), \quad (3)$$

where $f_{\mathbf{p}}^0 = (\exp[(\epsilon_{\mathbf{p}} - \mu)/k_B T] + 1)^{-1}$ is the equilibrium Fermi distribution, μ is the chemical potential, $\epsilon_{\mathbf{p}} = \mathbf{p}^2/2m$, and $f_{\mathbf{p}\sigma}(\mathbf{r}, t) = (\exp[(\epsilon_{\mathbf{p}} - \mu_\sigma(\mathbf{r}))/k_B T_\sigma(\mathbf{r})] + 1)^{-1}$ is the local equilibrium distribution, $\partial_\epsilon f_{\mathbf{p}}^0 = -f_{\mathbf{p}}^0(1 - f_{\mathbf{p}}^0)/k_B T$, and $\phi_{\mathbf{p}\sigma}$ is determined by solving the Boltzmann equation for the spin distribution $n_{\mathbf{p}s} = n_{\mathbf{p}+} - n_{\mathbf{p}-}$ in linear response,

$$\partial_\epsilon f_{\mathbf{p}}^0 \left(\frac{\epsilon_{\mathbf{p}} - w(T)}{k_B T} \right) \mathbf{v}_{\mathbf{p}} = \mathbf{C}_{\mathbf{p}}[\phi], \quad (4)$$

where $w(T) = \mu + Ts$ is the enthalpy per particle and s is the entropy per particle [13]. We defined $\phi_{\mathbf{p}s} \equiv k_B \phi_{\mathbf{p}} \cdot (-\nabla T_s)$, and expressed the linearized collision integral in the Boltzmann equation for the spin distribution as $(\partial n_{\mathbf{p}s}/\partial t)_{\text{coll}} \equiv \mathbf{C}_{\mathbf{p}}[\phi_{\mathbf{p}}] \cdot (-\nabla T_s)$. The spin current is given by

$$\mathbf{j}_s = - \int \frac{d^3 p}{(2\pi\hbar)^3} \partial_\epsilon f_{\mathbf{p}}^0 \mathbf{v}_{\mathbf{p}} \phi_{\mathbf{p}s}. \quad (5)$$

We solve Eq. (4) using the method described in Ref. [5]. Applying the temperature gradient along the x -axis, we parametrize the response by a power series,

$$\phi_{\mathbf{p}s}(a, T) = \left[b_0(a, T) + b_1(a, T) \left(\frac{\epsilon_{\mathbf{p}}}{k_B T} \right) \right] p_x (-k_B \partial_x T_s). \quad (6)$$

The coefficients b_0 and b_1 , determined by the approximate solution to Eq. (4), are given by

$$\begin{Bmatrix} b_0(a, T) \\ b_1(a, T) \end{Bmatrix} = \frac{3nl(T)}{C_{00}C_{11} - C_{01}^2} \begin{Bmatrix} -C_{01}(a, T) \\ C_{00}(a, T) \end{Bmatrix}, \quad (7)$$

where

$$l(T) = \frac{35}{4} \frac{f_{7/2}(z)}{f_{3/2}(z)} - \left(\frac{w(T)}{k_B T} \right)^2, \quad (8)$$

$z = e^{\mu/k_B T}$ is the fugacity, $f_n(z) = -\text{Li}_n(-z)$, $\text{Li}_n(z)$ are the polylogarithmic functions, and C_{nm} are the 2×2 matrix elements of the collision integral,

$$\begin{aligned} C_{nm} = & \frac{1}{k_B T} \int \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi\hbar)^6} \frac{|\mathbf{p}_1 - \mathbf{p}_2|}{m} f_{\mathbf{p}_1}^0 f_{\mathbf{p}_2}^0 \\ & \times \frac{1}{4} \int d\Omega_r (1 - f_{\mathbf{p}_3}^0)(1 - f_{\mathbf{p}_4}^0) \\ & \times \left\{ \frac{d\sigma_{++}}{d\Omega_r} \Delta_{++}[(\epsilon_{\mathbf{p}}/k_B T)^n \mathbf{p}] \cdot \Delta_{++}[(\epsilon_{\mathbf{p}}/k_B T)^m \mathbf{p}] \right. \\ & \left. + \frac{d\sigma_{+-}}{d\Omega_r} \Delta_{+-}[(\epsilon_{\mathbf{p}}/k_B T)^n \mathbf{p}] \cdot \Delta_{+-}[(\epsilon_{\mathbf{p}}/k_B T)^m \mathbf{p}] \right\}, \quad (9) \end{aligned}$$

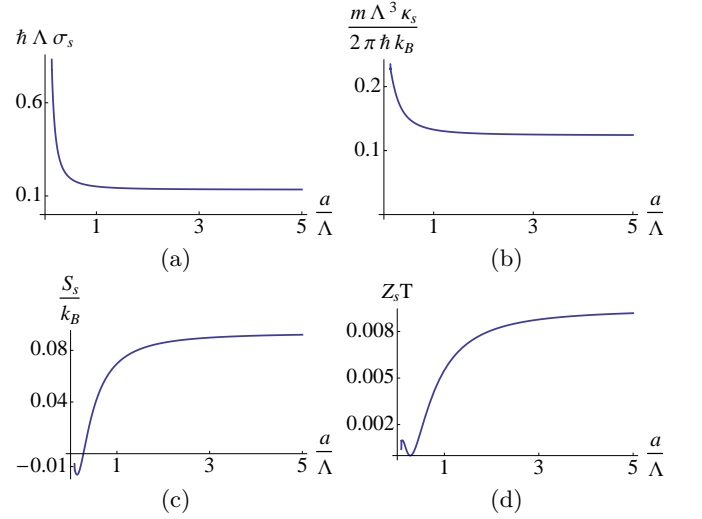


FIG. 4: Plots of the transport coefficients in the high-temperature limit, $T \gg T_F$, as a function of a/Λ . (a) The spin conductivity normalized as $\hbar\Lambda\sigma_s$, (b) the spin heat conductivity normalized as $m\Lambda^3\kappa_s/2\pi\hbar k_B$, (c) S_s/k_B and (d) $Z_s T$.

where we define $\Delta_{+-}[\phi_{\mathbf{p}}] = \phi_{\mathbf{p}_3} + \phi_{\mathbf{p}_4} - \phi_{\mathbf{p}_1} - \phi_{\mathbf{p}_2}$ and $\Delta_{+-}[\phi_{\mathbf{p}}] = \phi_{\mathbf{p}_3} - \phi_{\mathbf{p}_4} - \phi_{\mathbf{p}_1} + \phi_{\mathbf{p}_2}$ for an arbitrary function $\phi_{\mathbf{p}}$, and in the integrand momentum conservation is satisfied: $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_3 + \mathbf{p}_4$.

From Eq. (5) and Eq. (1), it follows that the Seebeck coefficient is given in terms of b_0, b_1 and the spin conductivity σ_s by

$$S_s(a, T) = \frac{n}{\sigma_s} \left[b_0(a, T) + b_1(a, T) \frac{w(T)}{k_B T} \right]. \quad (10)$$

The other transport coefficients are calculated similarly. In order to make the numerics more tractable, we have omitted the angular dependence in $d\sigma_{++}/d\Omega$. On the other hand, in the high-temperature limit, the integrals in Eq. (9) including the angular factor can be done analytically [14], which allows us to calculate the coefficients in the high-temperature limit. The result is shown in Fig. 4 plotted as a function of a/Λ , where $\Lambda = \sqrt{2\pi\hbar^2/mk_B T}$. In this limit, the temperature dependence of σ_s can be understood from classical considerations. The spin conductivity is approximately related to the inter-spin collision time τ_{+-} by $\sigma_s \propto n\tau_{+-}$ and $1/\tau_{+-} = n\bar{\sigma}_{+-}v_T$, where $\bar{\sigma}_{+-}$ is the inter-spin cross section and $v_T \propto 1/\Lambda$ is the average thermal velocity. In the limit $a/\Lambda \rightarrow 0$, $\bar{\sigma}_{+-} \propto a^2$, thus $\Lambda\sigma_s \propto (\Lambda/a)^2$, and when $a/\Lambda \rightarrow \infty$, $\bar{\sigma}_{+-} \propto \Lambda^2$ thus $\Lambda\sigma_s \propto 1$, in agreement with our result.

The behavior of the transport coefficients depends crucially on the shape of the perturbed spin distribution $\delta n_{\mathbf{p}s} = -\partial_\epsilon f_{\mathbf{p}}^0 \phi_{\mathbf{p}s}$ and the associated spin current density, which we plot for $k_F a = 10$ in Fig. 5. The positive (negative) parts of $\delta n_{\mathbf{p}\sigma}$ may be regarded as particle (holes) having group velocities $\pm \mathbf{p}/m$. Since

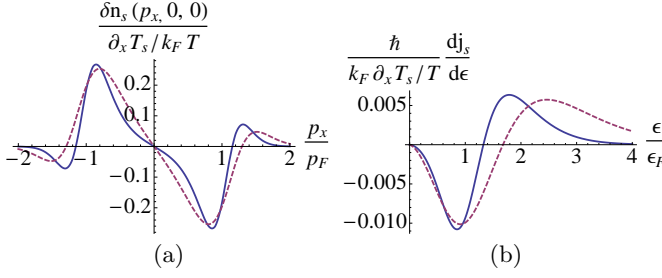


FIG. 5: A plot of (a) the perturbed spin distribution per spin temperature gradient, normalized as $\delta n_s / (-\partial_x T / k_F T)$ along the p_x -axis, and a plot of (b) the spin current density along the p_x -axis, per $d\epsilon$ per spin temperature gradient, normalized as $(dj_s / d\epsilon)(\hbar / k_F (\partial_x T_s / T))$, for $k_F a = 10$ and temperatures at which S_s is negative ($T = .3T_F$, blue thick line) and positive ($T = .5T_F$, purple dashed line).

$\delta n_{\mathbf{p}\sigma} = -\delta n_{\mathbf{p}-\sigma}$, every spin up particle is matched with a spin down hole with the same momentum, resulting in the spin current. Thus, the sign of S_s is determined by the relative number of particles or holes induced in response to the spin temperature gradient.

Next, we give a criterion that determines the sign of the spin-Seebeck coefficient and show that it does not depend on the form of the intra-spin scattering at all. First, we note that in our solution given by Eq. (7), we always have $b_0 < 0$ and $b_1 > 0$ because $l(T) > 0$, $C_{nm} > 0$, and $\det \hat{C} > 0$ [15], which implies $b_0 < 0$ and $b_1 > 0$. Thus for positive momenta, $b_1(b_0)$ corresponds to particles (holes) created above (below) the Fermi surface. Inspecting Eq. (10), we find that the criteria to have $S_s \leq 0$ is

$$\left| \frac{b_0}{b_1} \right| = \left| \frac{C_{01}}{C_{00}} \right| \geq \frac{w}{k_B T}, \quad (11)$$

and the opposite inequality for $S_s > 0$. Thus S_s is positive when b_1 becomes large enough to violate Eq. (11) [16]. Furthermore, it turns out that the only integral in Eq. (9) containing intra-spin scattering that is non-vanishing is C_{11} , which does not enter in Eq. (11).

The temperature dependence of b_0, b_1 follows from the temperature dependence of the collision matrix elements in Eq. (9), which are given by integrals nonvanishing only for $p_r \sim \sqrt{4\pi\hbar}/\Lambda$. Therefore, it is useful to express the differential cross section Eq. (2) in terms of the rescaled momentum $\tilde{p}_r = (\Lambda/\sqrt{4\pi\hbar})p_r$,

$$\frac{d\sigma_{+-}}{d\Omega} = \frac{1}{k_F^2} \frac{(k_F a)^2}{1 + (k_F a)^2 (T/T_F) \tilde{p}_r^2}, \quad (12)$$

From Eq. (12) we see that the change in the sign of S_s is related to the crossover from hard-sphere scattering, $d\sigma/d\Omega \simeq a^2$ when $k_F a \ll 1$ or $T/T_F \ll 1$, to momentum-dependent scattering, $d\sigma/d\Omega \sim \tilde{p}_r^{-2}$ when $k_F a \simeq 1$ and $T \simeq T_F$.

Discussions and outlook.— We note that as the temperature is lowered, one expects to enter the Fermi-liquid regime, for $T_c < T \ll T_F$, where T_c is temperature for the superfluid transition, and one should use Fermi-liquid scattering amplitudes in the collision integral [8]. Near and above T_c , one should also take into account effects of pairing correlations on the inter-spin interaction [17]. Both these regimes can be analyzed with the formalism presented in this paper, and will be relegated to future work.

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- ¹⁴ Specifically, C_{nm} can be expressed in terms of incomplete Gamma functions.
- ¹⁵ The eigenvalues of the collision matrix are negative because they correspond to relaxation times, and therefore, for the 2×2 collision matrix, the determinant must be positive.
- ¹⁶ Note that the Seebeck coefficient vanishes in a simple relaxation time approximation given by $\phi_{\mathbf{p}s} = \tau_s(w - \epsilon_{\mathbf{p}})v_x \partial_x T_s / T$, corresponding to the case $|b_0/b_1| = w/k_B T$ in Eq. (11).
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